

## 1.8. Validity Counterexamples, And an Informal Test of Validity

**1. Validity Counterexamples.** Understanding logic as the study of **validity** – whether the conclusion of an argument **follows from** its premise(s) – we begin developing a test of validity. In its first form this test will remain informal and intuitive; but it will contain the elements essential to later, more sophisticated tests. Central to all these is a concept hinted at in previous examples of invalid arguments.

Recall that we judged Argument B invalid because it seems possible for the premises of B to be true while the conclusion is false.

### Argument B

$$1+1=2$$

$$2+2=4$$

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(so,) The first U.S. president was born in Boston.

Such a case reveals the invalidity of an argument because a *valid* argument should be immune to precisely this possibility. With a valid argument true premises are accompanied by true conclusion *without exception*, so there's no *possible* way of having true premises without a true conclusion. If there *is* a possible way for Argument B to have true premises but false conclusion, then Argument B doesn't fit the definition of a "valid argument".

We call such a possible situation a **validity counterexample** (or "counterexample," for short).

A **validity counterexample** for an argument is a possible situation where the premises of the argument are all *true*, but the conclusion is *false*.

For Argument B, the actual world – where  $1+1$  does indeed equal 2, and  $2+2$  equals 4, but the first U.S. president was not born in Boston – served as a validity counterexample. But we saw that with some arguments we need to stretch our imaginations to find a validity counterexample. Argument D illustrates this.

### Argument D

$$1+1=2$$

$$2+2=4$$

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(so,) The first U.S. president was born in Westmoreland County, Virginia.

In the actual world, the premises and conclusion of D are all true. But that isn't enough to make D valid – since it's still *possible* for the argument to have true premises with a false conclusion. With just a bit of imagination we described such a possibility: a situation where John Adams won the first U.S. election, while the mathematical facts remained the same. That situation would qualify as a validity counterexample for Argument D – establishing that the argument is *invalid*.

Just one validity counterexample is sufficient to prove an argument invalid. Indeed, that point was made clear by one of our alternate definitions of “valid argument” in the last section.

**Valid argument:** an argument where it's impossible to have true premises without having a true conclusion.

If we can show, for a given argument, that it *is* possible for it to have true premises without a true conclusion, we've shown that the argument falls short of being a valid argument.

For that reason the search for validity counterexamples takes center stage when testing an argument for validity. Roughly speaking: when testing an argument for validity we **try to think of a validity counterexample** for that argument. If we succeed in thinking up such a possibility, we know that the argument is invalid – and if invalid, unconvincing.

We won't rest content with this seat-of-the-pants, imagination-based test of validity. But already it provides an opportunity to rehearse the core concepts of **validity** and **validity counterexamples**, before grappling with more advanced tests. For even in those later tests, those two notions remain central.

**2. Validity Counterexample Examples.** The following simple argument supplies a bit of practice in picking out validity counterexamples.

**1. The first 43 presidents of the US have been white men.**

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**∴ All presidents of the US (past, present, future) are white men.**

We walk through a series of possibilities, asking for each whether it qualifies as a validity counterexample for this argument. First, consider Situation A.

**Situation A:** The first 43 US presidents are white men; after that they're men and women of various races.

In Situation A the premise of our argument would be true. But the conclusion of the argument – that “All presidents of the US (past, present, future) are white men” – is certainly false in A. Since Situation A makes all the premises of the argument true (all *one* of them) while making the conclusion false, Situation A qualifies as a **validity counterexample** for the argument.

Argument	Situation A
<b>1. The first 43 presidents of the US have been white men.</b> <hr/>	<b>TRUE</b>
<b>∴ All presidents of the US (past, present, future) are white men.</b>	<b>FALSE</b>

Thanks to Situation A, we know this argument is **invalid**: true premises do *not* guarantee a true conclusion here, so the conclusion does *not* follow from the premise. (Equivalently: the premises don't **entail** the conclusion.)

For a bit more practice, we continue with Possible Situation B.

**Situation B:** The US ends up having 102 presidents in all, and they're all white men.

In B the premise of the argument is **true**: if all 102 presidents are white men, then certainly the first 43 are. And here the conclusion is **true** as well: all the US presidents ever are white men.

Argument	Situation B
<b>1. The first 43 presidents of the US have been white men.</b>	<b>TRUE</b>
<hr/> <b>∴ All presidents of the US (past, present, future) are white men.</b>	<b>TRUE</b>

Situation B is **not** a validity counterexample for this argument. This situation tells us **nothing** about the validity of the argument.

How about Possible Situation C?

**Situation C:** Every president, from the first to the last, is a Hispanic man.

In such a situation the premise of the argument would be **false**: here it is not true that the first 43 US presidents are white men. The conclusion would be **false** in C as well.

Argument	Situation C
1. The first 43 presidents of the US have been white men.	FALSE
∴ All presidents of the US (past, present, future) are white men.	FALSE

Situation C does **not** qualify as a validity counterexample for this argument. C tells us **nothing** about this argument's validity.

We said that Situations B and C tell us nothing about the validity of the argument. Here's why.

Situation A already established that this argument is invalid. But Situation B shows us something important about an invalid argument: **an invalid argument can** (by lucky coincidence) **have true premises and true conclusion**.

Argument	Situation B
1. The first 43 presidents of the US have been white men.	TRUE
∴ All presidents of the US (past, present, future) are white men.	TRUE

Of course a **valid** argument can also have true premises and conclusion – as this example shows.

**Valid Argument:**

- 1. George Washington was the first US president.**
  - 2. George Washington was born in Virginia**
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**∴ The first US president was born in Virginia.**

In the actual world, for instance, the premises and conclusion of this valid argument are true.

**Having true premises and true conclusion is something both valid *and* invalid arguments can do.** So a situation like that – where the argument's premises and conclusion are all true – is no help in settling whether an argument is valid or invalid.

In Situation C the above invalid argument had false premises and a false conclusion.

Argument	Situation C
<b>1. The first 43 presidents of the US have been white men.</b> <hr/>	<b>FALSE</b>
<b>∴ All presidents of the US (past, present, future) are white men.</b>	<b>FALSE</b>

But that can happen to a valid argument as well.

**Valid Argument:**

- 1. Benjamin Franklin was the first US president.**
  - 2. Benjamin Franklin was born in Florida**
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**∴ The first US president was born in Florida.**

This argument is valid, because if the premises **were** both true, the conclusion would have to be true as well. Of course, in the actual world both the premises and conclusion are **false**. So: a valid argument can have false premises and conclusion in a certain situation.

Here again, **having false premises and false conclusion in a given situation is something both valid and invalid arguments can do**. So finding such a situation – where an argument's premises and conclusion are both false – tells us nothing about whether that argument is valid.

As single situations go, the **only** case that tells us anything about an argument's validity is a situation where the argument has true premises and a false conclusion – a **validity counterexample**. For that's the one sort of situation a valid argument will never find itself in.

So if, by scouring the world before us or any other possibilities the imagination can dream up, we find such a situation for a given argument, we know for certain that the argument is invalid. That's why the search for validity counterexamples plays such a central role in tests of validity.

### Summary: Validity Counterexamples

- A **validity counterexample** for an argument is a **possible situation where** the argument has **true premises** but a **false conclusion**.
- Finding a validity counterexample for an argument establishes that the argument is **invalid**. (As single situations go, no other kind of possible situation tells us *anything* about whether an argument is valid or invalid.)